ABSTRACT

The development of effective anti-collision warning radars is a key step in the technological initiatives promoted to improve road safety, and efficiency of transport systems. The ultimate objective remains crash prevention, through an accurate knowledge of the location, speed, acceleration or deceleration of the nearby vehicles. Automotive radars provide an automatic vision of the environment where the vehicle is moving, to draw the information required for guaranteeing, as much as possible, safety for the vehicle and the passengers on board. About the expected performance, the target of an automotive radar is to have a high detection capability but also a low false detection rate; the analysis shown in this Chapter will be mainly focused on such figures.

Traditionally, Short Range Radars (SRR) and Long Range Radars (LRR) have been used. Classic LRR solutions employ Frequency Modulation - Continuous Wave transmission, that is very simple to implement but also prone to interfering signals produced by neighboring radars of the same type. To overcome this limitation, radars employing Spread Spectrum methods in Direct Sequence configuration have been introduced. All spread spectrum systems make use of pseudorandom codes, that determine the frequency spectrum occupied by the output signal, and control the spreading pattern of the system. The accuracy in the distance evaluation (that is of course an important result of radar detection) depends on the auto-correlation properties of the spreading sequences associated to each radar equipment; on the other hand, the ability to reject the interference due the other users depends on the cross-correlation properties of the whole set of sequences adopted. Another important aspect concerns the number of signatures available; in traditional techniques, this number depends on the sequence length, and the latter cannot be arbitrarily large, because of the constraints on the bandwidth. To improve these features and overcome existing limitations, the adoption of innovative sets of spreading sequences, such as chaotic and De Bruijn sequences, is investigated and evaluated as an alternative to more conventional solutions. This Chapter provides a brief introduction to Spread Spectrum radar applications in the automotive scenario, by highlighting the evolution from traditional to more advanced solutions, with a specific focus on the selection of innovative sets of spreading codes, and the theoretical evaluation of the performance obtainable.
INTRODUCTION

Chaos theory, a branch of the theory about nonlinear systems, has been intensively studied in the past decades. Initially, it was investigated by researchers with strong mathematical background, rather than circuit-designers or electronic engineers. This was mainly due to the fact that circuit design and implementation could not match up with the mathematical equations describing chaotic systems, because of technical and practical implementation problems. With the advances in circuit technology and digital signal processing, the exploitation of chaotic phenomena for a number of telecommunication applications (Sobhy and Shehata, 2001), and in daily real-life engineering products, became possible. Nowadays, more and more systems and applications may benefit from using the chaotic dynamic behavior.

A chaotic system is particularly sensitive to environment changes and highly dependent on the initial conditions settings. Even a small difference in the initial conditions produces a very different chaotic signal, after a short time period. Therefore, it is possible to generate a large number of chaotic signals, even with a very simple dynamic deterministic equation. Such a behavior makes chaotic systems highly unpredictable, and more similar to an ideal white noise source than any other pseudo-noise (PN) source currently in use. This resemblance to white noise and, more generally, to uncorrelated sources, reflects on the very favorable auto- and cross-correlation properties that chaotic signals exhibit when used as spreading sequences, in spread spectrum (SS) systems. Such correlation properties can make these signals preferable against competing solutions, like those based on Gold codes (Sivanesan and Beaulieu, 2001). Moreover, as said above, the number of chaotic signals, for a prefixed bandwidth occupation and correlation properties, can be very large, thus removing the problem on the number of codes available.

Spread spectrum techniques may be defined as methods by which the signal energy generated in a particular bandwidth is deliberately spread in the frequency domain, thus resulting in a signal with a wider bandwidth. Such techniques have become more and more popular, especially as the available frequency spectrum is becoming more and more crammed. Although spread spectrum systems are more complex and expensive to implement than conventional systems, they have many advantages. The major one is the inherent signal security they can provide. Unlike conventional systems, it is extremely difficult to eavesdrop on a conversation that takes place over a spread spectrum system. Conventional systems try to allocate as much information into as small a bandwidth as possible. These systems, however, can easily be jammed by a high-power jamming signal at a frequency that covers the frequency band of the particular system. They also output a relatively high power to the antenna.

Spread spectrum systems, on the contrary, spread the signal over as wide a bandwidth as possible; they also try to hide the transmitted signal as close to the background noise as possible. This makes the communication very difficult to intercept, in the frequency domain: it cannot be easily tracked, and it is more difficult to jam. All spread spectrum systems make use of pseudorandom code generators. Such codes are referred to as pseudorandom noise codes (PN codes). Each code in a given set is used to
set the frequency spectrum that the output signal will occupy. It also determines and controls the spreading pattern of the system: by using a carefully designed and selected spreading sequence set, which benefits from good correlation properties, it is possible to implement multiple access technologies, meaning multi-user transmission at the same time, in the same frequency band.

Chaotic signals are highly unpredictable, similar to an ideal white noise source, and very easy to generate and control. The adoption of chaotic radars for automotive applications has been already suggested (Sobhy and Shehata, 2001; Sivanesan and Beaulieu, 2001; Gambi, 2008); however, the results presented were quite preliminary. In this Chapter, a further contribution is discussed, through theoretical and numerical analysis, by investigating the performance, in terms of detection capability and false detection rates, of Direct Sequence Spread Spectrum (DS-SS) radars based on a class of chaotic sequences. A comparison with a solution based on classical Gold codes is developed, by considering typical, although simplified, road environments. The improvement that is achievable by using chaotic sequences is well appreciated when focusing on the worst operation conditions, with the automotive radar subject to blinding from other radars. Under these very severe conditions, chaotic sequences can permit to increase the correct detection probability (CDP), and to reduce the false detection probability (FDP).

**Automotive Radar Systems**

Improving road safety has become one of the most important priorities for any government policy all over the world. Just to mention the position of the European Commission, that inspires the policies of most European countries, a fundamental objective is to “improve safety, security, comfort and efficiency in all transport models (...) focusing on advanced pilot/driver assistance systems, in support of vision, alertness, manoeuvring, automated driving compliance with the regulations, etc”. According to this orientation, many research programs, like e-Safety (ECC, 2003), have been developed, seeing the participation of academic and industrial partners. The situation is not different outside Europe (SAFETEA-LU, 2007; ASV, 2007). In the USA, the National Highway Traffic Safety Administration (NHTSA) has reported that there are 6.2 million crashes annually resulting in more than 43,000 fatalities and a cost to society of more than $230 billion (Wenger, 1998). Half of these fatalities occur in cases of vehicles leaving a road and passing through intersections. In addition, injuries and damages from non-fatal accidents lead to significant costs in terms of healthcare and property. Safety systems traditionally applied by automakers are based on individual vehicle implementations, such as airbags and anti-lock brakes. However, despite some substantial initial improvements to road safety provided by these solutions, single-vehicle systems have not been able to further reduce fatalities or injuries significantly. The number of death events has kept gradually increasing in recent years, even though most of the nations have strong laws and policies supporting highway safety. Road accidents are often caused by driver carelessness or ignorance, simple misconduct, or lack of experience. In some instances, dangers due to severe weather and road conditions, and consequent obstruction of view, are also responsible.
It is extremely difficult to eliminate these human factors due to the inherent limits of human sensing and reaction capabilities; most people all over the world give their lives in transportation related accidents, more than any other single cause of death. In this context, the Wireless Access in Vehicular Environments (WAVE) technology is a revolutionary solution for vehicle safety enhancement, providing drivers with early warning, perceive and assistance. It is an extension of human natural sensing, and enables tele-sensing of vehicles. Working as probes, vehicles report timely traffic and road condition information to transportation agencies; the same information is thereafter shared by a large community of users. The standardization activity of the WAVE system, through the initiatives of the Intelligent Transportation System (ITS) Committee of the IEEE’s Vehicular Technology Society, that led to several recommendations published from 2006, introduces the next generation, dedicated, short-range communications technology, which provides high-speed vehicle-to-vehicle and vehicle-to-infrastructure data transmission, and has major applications in ITS, vehicle safety services, and Internet access.

The final WAVE architecture provides specific protocols for real-time safety message delivery in the vehicular environment, to effectively support road safety. Once in place (cars with WAVE started come off the assembly line in 2011), the system will provide driver alerts, including visual, audible, and tactile warnings. A warning of an impending accident will be fed to pre-crash systems, like those now found only on some luxury cars, to perform specific actions, like pre-tensing seat belts, prepare brakes for an emergency stop, and tilt reclined seats upright. However, the most important objective remains to prevent the crash through an accurate knowledge of the location, speed, acceleration or deceleration of the surrounding vehicles. The auto-industry is pouring a lot of efforts and money in these projects, and the expectation level is very high.

Automotive radars permit an automatic vision of the environment where the vehicle is moving, and, from that, to draw the information required to perform some actions that guarantee, as much as possible, safety for the vehicle and the passengers onboard. In designing radar systems for automotive applications, it is necessary to comply with national constraints on the usable frequencies and allowed power levels. In particular, several recommendations have been already issued by national and international agencies (ETSI, 2004; ETSI, 2005; FCC, 2006; ETSI, 2004), that put strict limits on the power and the spectral occupancy allowed to radar equipments. These are necessary for electromagnetic compatibility with other systems, and to ensure the absence of any effect on the human health. Moreover, it is mandatory to limit the costs: only cost-efficient systems will be accepted by the market for a widespread use, which in turn is a prerequisite for effective public benefit. Another important technological and commercial requirement concerns the need to keep small sensor sizes, due to their location in the vehicle frame.

From the performance viewpoint, the target of an automotive radar is to have a high detection capability but also a low false detection rate; however, other issues, like installation, maintenance and auto-calibration, must be taken under control as well.
Two kinds of radar have been considered for automotive applications up to now: Short Range Radars (SRR) and Long Range Radars (LRR). In the framework of European regulations, SRRs operate in the ISM band of 24 GHz or in the 77 GHz to 81 GHz band, with a visibility range from 0 to 30 meters, approximately. They are used for a number of applications, to enhance the active and passive safety for all kind of road users. Applications that enhance passive safety include obstacle detection, collision warning, lane departure warning, lane change aid, spot detection, parking aid and airbag arming, as depicted in Fig.1. The grouping of these functions is referred to in the literature as a “safety belt” for cars. The SRR is a combination of two functions: it allows a precise speed measurement combined with an accurate radial range information. To obtain the required resolution, in the order of approximately 5 to 10 centimeters, the SRR needs a large bandwidth of 4 GHz, for the range measurement.

LRRs operate between 76 GHz and 77 GHz with a maximum bandwidth of 1 GHz. They allow an operating range of approximately 150 meters and are used at vehicle velocities not below 30 Km/h. One or multiple narrow lobes control and scan the driving path in front of the car, to determine the distance to the vehicle running ahead, thus maintaining a constant minimum safety gap, and allowing an Adaptive Cruise Control (ACC).

Fig.1. Possible applications for automotive radars.

Limiting our focus on LRRs, classic implementations employ signal processing techniques based on Frequency Modulation-Continuous Wave (FM-CW) transmission, and a 1 GHz bandwidth at the most. FM-CW systems are very simple to implement but their performance can be strongly degraded because of the interfering signals produced by neighboring radars of the same type. To overcome this limitation, radars employing Spread Spectrum Pseudo Noise methods in Direct Sequence configuration have been proposed.

The specific PN sequence is the radar “signature”, which allows to compute the distance of a vehicle by selecting its echo among a very large number of echoes due to other vehicles, but also to the signals coming from other radars operating in the same frequency bandwidth. The accuracy in the distance evaluation depends on the auto-correlation properties of each sequence; the ability to reject the interference due to the other users.
depends on the cross-correlation properties of the whole set of sequences adopted. Another important aspect concerns the number of signatures available; in case of PN sequences, this depends on the sequence length, and there are relatively small sets of signatures available. For these reasons, it seems meaningful to explore possible applicability of chaotic signals in place of more conventional solutions, to increase the number of available signatures and maintain, at the same time, the required correlation properties.

This Chapter provides insights about the possibility of adopting chaotic sequences as radar signatures in automotive applications, to overcome the limits of PN sequences discussed above. Some preliminary evaluations are also provided for an alternative set of spreading codes, i.e. the De Bruijn binary sequences, that up to now have been mainly applied in cryptographic contexts. The performance obtainable are shown and discussed, in different application scenarios; open issues and problems are also evidenced.

SPREADING SEQUENCES FOR DS-SS SYSTEMS

In this section, a brief overview of the sequences considered as spreading codes is provided.

Gold Sequences

Binary Gold sequences are a well known set of PN sequences, also used in wireless mobile communications (WCDMA) as scrambling codes, either in uplink (for separating the mobile stations) or in downlink (for separating the cells). They are obtained by combining two maximum-length shift register sequences (M-sequences) (Proakis, 1995) of length \( N = 2^m - 1 \), where \( m \) is the size of the shift-register. The Linear Feedback Shift Registers (LFSR) and the connections selected to generate \( m \)-sequences of length 32, 64 and 127, respectively, are illustrated in Fig. 2.

Gold sequences exhibit a three-valued cross-correlation function, with values \{-1, -t(m), t(m) - 2\}, where \( t(m) \) is equal to:

\[
t(m) = \begin{cases} 
2^{(m+1)/2} + 1 & \text{(for odd and even } m \text{ respectively)} \\
2^{(m+2)/2} + 1 & 
\end{cases}
\]

The characteristic three-valued cross-correlation function of a pair of Gold sequences of length 63 is shown in Fig.3.
Fig. 2. Gold sequences generators.

Fig. 3. Cross-correlation function of a pair of Gold sequences of length 63.

Also the auto-correlation function is a three-valued function, with values \( \{N, -t(m), t(m) - 2\} \), as illustrated in Fig. 4.

The number of sequences, for a given \( N \), is \( N + 2 \). Since the sequence length equals the Spreading Factor (SF), for an assigned SF, the number of spreading sequences is \( SF + 2 \).

Gold codes show good auto- and cross-correlation properties, particularly for large values of \( m \). These properties, however, can be outperformed using chaotic sequences, whose rationale is discussed in the following.
Chaotic Sequences

Chaotic signals have been recently valued in CDMA systems, mostly because of their favorable correlation properties (Rovatti, 2004; Chiaraluce, 2002; Rovatti et al., 2004). The “perfect” random generator would be an ideal white noise source. A number of physical processes, relying on phenomena that produce random noise at a microscopic scale, could be used, in principle, to mimic a completely uncorrelated random source. Anyway, these natural phenomena are generally hard to control and/or to model by a mathematical approach. For this reason, physical sources are often replaced by pseudo-noise sources that, being necessarily based on finite memory algorithms, exhibit periodic behaviors and generate correlated examples. Therefore, their behavior may be significantly far from the ideal one. Gold sequences are examples of signals produced by pseudo-noise sources.

A significant advance towards the idea of realizing an artificial source with features very similar to the ideal ones is represented by chaotic generators. They have a deterministic basis, like pseudo-noise sources; their peculiar feature is to exploit a simple basic rule, in an iterative process, that rapidly produces a very complex and quite unpredictable evolution. As a consequence, signals obtained by chaotic sources are very irregular, not periodic, and extremely sensitive to the initial conditions: two identical systems, apparently starting at identical conditions, will end up with totally different outputs. These behaviors correspond to the commonly recognized concept of chaos and are very appealing in the perspective of designing good spreading sequences.

Chaotic sources are obviously available also in nature, but the advantage of an artificial apparatus is in the possibility to obtain the desired chaotic behavior through elementary mathematical rules, that are quite easy to control (at least in the generation process) and to implement by electronic circuits. As a matter of fact, a simple Bernoulli shift map, that is a piece-wise affine transformation, can behave like an ideal binary random generator and can be implemented with a single stage of a pipeline analogue-to-digital converter.
In our study, we used the chaotic evolution resulting from the solutions of the following, well-known, Lorenz system of equations:

\[
\begin{align*}
\frac{du}{dt} &= \sigma \cdot (v - u) \\
\frac{dv}{dt} &= r \cdot u - v - 20 \cdot u \cdot w \\
\frac{dw}{dt} &= 5 \cdot u \cdot v - b \cdot w 
\end{align*}
\] (2)

In the system, \(\sigma\), \(r\) and \(b\) are suitably fixed parameters (a typical choice assumes \(\sigma = 16\), \(r = 45.6\) and \(b = 4\)) while \(u(t)\), \(v(t)\) and \(w(t)\) are the system states; each state variable can be used for providing the time evolution of a chaotic spreading signal. The Lorenz system is solved starting from assigned initial conditions (for example \(u(0) = 0.82\), \(v(0) = 0.63\) and \(w(0) = 0.74\)); the result is an unpredictable set of not periodic functions, each one showing features very similar to white noise and therefore with excellent correlation properties. The time evolution of the \(u(t)\) state, with the following parameters, is shown in Fig.5.

\[
\begin{align*}
\sigma &= 16 \\
r &= 45.6 \\
b &= 4 \\
u_0 &= 0.82 \\
v_0 &= 0.63 \\
w_0 &= 0.74
\end{align*}
\] (3)

For a given set of initial conditions, different chaotic spreading signals can be obtained by generating a long chaotic evolution and then braking it into a desired number of segments (each segment represents a single spreading signal). This is the process applied in our study. Alternatively, different spreading signals may be generated by solving the Lorenz system with different initial conditions.

Fig.5. Time evolution of the \(u(t)\) state.

The solutions to the Lorenz system are real functions. In this sense, they can be seen as multilevel signals. Their amplitudes can be quantized in order to obtain a fully digital sequence. Sometimes, chaotic sequences are obtained directly in binary form (Callegari, 2005).
Such an approach may provide advantages in the implementation, by smoothing some practical difficulties about the repeatability of chaotic maps. Anyway, the conclusions drawn for Lorenz signals can be considered valid, with a good approximation, for the binary chaotic sequences too. An interesting property of chaotic generation is the possibility to increase the number of signals with specified correlation properties. This feature is useful in the automotive context, where the need of correctly discriminating a (possibly very) large number of users is a distinctive property.

A means for specifying the desired correlation properties of the set of spreading codes relies in fixing suitable thresholds for their auto- and cross-correlation (AC, CC) values. Noting by $c^i_n$ the n-th chip of the j-th spreading sequence, the correlation at a shift $\tau$ between the i-th and the j-th sequences is defined as:

$$P_j(\tau) = \sum_{n=0}^{N-1} e^i_n e^j_n$$

where $N$ is the sequence length. The normalized maximum autocorrelation value, for $\tau \neq 0$ results in:

$$P_{A} = \max_{\tau \neq 0} \frac{1}{P_j(0)} \left| P_j(\tau) \right|$$

whilst the maximum cross-correlation between the j-th sequence and all the other sequences in the set is:

$$P_{C} = \max_{\tau} \frac{1}{P_j(0)} \left| P_j(\tau) \right| \quad i \neq j \quad [1..S]$$

Following (Morantes, 1998), from a set of S chaotic signals randomly generated, a first selection is made by eliminating those sequences with $P_A$ greater than a threshold $\varepsilon_1$; then, if applicable, the selection is refined by eliminating those sequences with $P_C$ greater than a threshold $\varepsilon_2$. An example based on the first selection strategy, with a variable $\varepsilon_1$, is shown in Table 1.

Table 1. Number of selected sequences fixing a threshold on the auto-correlation.

<table>
<thead>
<tr>
<th>$\varepsilon_1$</th>
<th>Number of sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0.62$</td>
<td>400</td>
</tr>
<tr>
<td>$0.61 + 0.55$</td>
<td>400 + 396</td>
</tr>
<tr>
<td>$0.545 + 0.485$</td>
<td>395 + 382</td>
</tr>
<tr>
<td>$0.48 + 0.42$</td>
<td>380 + 337</td>
</tr>
<tr>
<td>$0.419 + 0.355$</td>
<td>335 + 229</td>
</tr>
<tr>
<td>$0.354 + 0.291$</td>
<td>229 + 107</td>
</tr>
<tr>
<td>$0.29 + 0.226$</td>
<td>107 + 19</td>
</tr>
<tr>
<td>$0.225 + 0.2$</td>
<td>18 + 5</td>
</tr>
</tbody>
</table>
The starting set contains $S = 400$ chaotic sequences, with $N = 31$. All the sequences have $P_A$ smaller than $\varepsilon_1 = 0.62$, but 229 out of them, for example, have $P_A$ smaller than $\varepsilon_1 = 0.355$; only 107 sequences exhibit a $P_A$ smaller than $\varepsilon_1 = 0.291$. When the second selection criterion is applied, the number of sequences is, obviously, further reduced. An example is reported in Table 2, where the same sequences with $P_A \leq 0.291$, selected before, have been tested for their cross-correlation value. All the 107 sequences have $P_C$ smaller than $\varepsilon_2 = 0.81$, but only 7 out of them, for instance, exhibit a $P_C \leq 0.55$. It should be noted, however, that such a result depends on the relatively small size of the starting set ($S = 400$); by increasing the size of the starting set, larger amounts of chaotic spreading sequences with prefixed correlation properties should be found.

Table 2. Number of selected sequences fixing a threshold on the cross-correlation ($\varepsilon_1 = 0.291$).

<table>
<thead>
<tr>
<th>$\varepsilon_1$</th>
<th>Number of sequences</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0.81$</td>
<td>107</td>
</tr>
<tr>
<td>$0.8 \div 0.745$</td>
<td>107 + 106</td>
</tr>
<tr>
<td>$0.74 \div 0.678$</td>
<td>106 + 92</td>
</tr>
<tr>
<td>$0.677 \div 0.613$</td>
<td>92 + 41</td>
</tr>
<tr>
<td>$0.612 \div 0.55$</td>
<td>41 + 7</td>
</tr>
<tr>
<td>$0.545 \div 0.485$</td>
<td>7 + 3</td>
</tr>
<tr>
<td>$0.48 \div 0.42$</td>
<td>3 + 1</td>
</tr>
<tr>
<td>$0.415$</td>
<td>1</td>
</tr>
</tbody>
</table>

In the case of Lorenz chaotic signals, the spreading factor does not result “naturally” from the solution of the Lorenz system. For a given set of parameters and initial conditions, signal evolution depends on the discretization step $h$ (for computational purposes, the Lorenz system is in fact approximated by an equivalent discrete form) and on the sampling period $T_k$:

\[
\begin{align*}
    u_{t+1} &= u_t + b \cdot \sigma \cdot (v_t - u_t) \\
    v_{t+1} &= v_t + b \cdot (r \cdot u_t - v_t - 20 \cdot u_t \cdot w_t) \\
    w_{t+1} &= w_t + b \cdot (5 \cdot u_t \cdot v_t - b \cdot w_t)
\end{align*}
\]  

(7)

A possible procedure consists in fixing the value of $T_k$ and then assuming a number of chaotic samples that ensure (approximately) the same bandwidth expansion factor of the competing Gold sequences. Such a strategy, that implies to develop a parallel analysis in the frequency domain, seems also the most proper one for ensuring a fair performance comparison. Even more important, the selected chaotic evolutions exhibit auto- and cross-correlations with reduced dispersion around the average value: at a parity of the number of sequences, the auto- and cross-correlations of chaotic signals have, typically, smaller standard deviations (Chiaraluce, 2002). As performance depends on the variance of the disturbance, it is reasonable to expect this may have a positive impact on the performance results.
Binary De Bruijn Sequences

Binary De Bruijn sequences (Mayhew, 2000; Mayhew, 2007; Andrenacci, 2010) are nonlinear shift register sequences with maximal period, $N = 2^n$; $n$ is called the span of the sequence, i.e. the number of stages in the generating shift register (Jansen, 1991). They may be generated by $n$-stage nonlinear feedback shift registers, as they include the all zero $n$-tuple, which cannot be included if a linear shift register is adopted for the generation process. In the binary case, the total number of distinct sequences of span $n$ is $2^{2^{n-1} - n}$. The states $S_0$, $S_1$, . . . , $S_{2^n-1}$ of a span $n$ De Bruijn sequence are exactly the $2^n$ different binary $n$-tuples; when viewed cyclically, a De Bruijn sequence of length $2^n$ contains each binary $n$-tuple exactly once over a period. Let us consider the simple case of $n = 3$. For $n = 3$, two distinct De Bruijn sequences exist. Each sequence contains all the possible binary 3-tuples (000, 001, 010, 011, 100, 101, 110, 111) exactly once over a period of length $2^3 = 8$. The sequences are:

\[ s_1 = 000\text{10111} \]
\[ s_2 = 111\text{0100} \]

De Bruijn sequences have many desirable properties, such as long period and low predictability, and can be used as stream ciphers in cryptographic applications. As discussed by Mayhew in (Spinsante, 2011), a De Bruijn sequence may also be obtained by adding a single zero to the longest run of zeros in an M-sequence, thus obtaining a so called primitive De Bruijn sequence; at the same time, a modified De Bruijn sequence is obtained when removing a zero from the all zero $n$-tuple of a De Bruijn one. However, not all the modified De Bruijn sequences are M-sequences: the latter represent the linear subset of the modified De Bruijn sequences family.

Being De Bruijn sequences maximal period sequences, their length is always an even number, and a power of 2 in the binary case. When comparing the cardinality of a set of binary De Bruijn sequences, for a given value of the span $n$, to the cardinality of other families of sequences of corresponding length (such as Gold or M-sequences), it is clear that, at least theoretically, the exponential law in the De Bruijn case could make it possible to enlarge the set of potential users in a spread spectrum system, in case also the other necessary properties are verified. As an example, for $n=10$ there exist up to 60 binary M-sequences of length 1023, up to 1025 binary Gold sequences of length 1023, and up to $2^{502}$ binary De Bruijn sequences, that is an extremely huge amount. Of course, not all the $2^{502}$ sequences may anyway be suitable for application in a multi-user system. In any case, even if strict selection criteria are applied, it is reasonable to expect that a quite huge amount of sequences may be extracted from the entire family.

Several results are known about the correlation properties of binary De Bruijn sequences. The autocorrelation $P_A(k)$ of a binary De Bruijn sequence $c = (c_0, c_1, \ldots, c_{L-1})$ of length $L = 2^n$, for a given shift $k$, may assume only a set of given values:
\[ P_a(k) = 2^n \], \text{for} \ k = 0 \\
P_a(k) = 0, \text{for} \ 1 \leq |k| \leq n - 1 \\
P_a(k) \neq 0, \text{for} \ |k| = n \]

(8)

The second property implies that as long as the span of the sequence increases, there are more null samples of the autocorrelation sidelobes (i.e. the values assumed by \( P_a(k) \) for \( k \neq 0 \)) that provide a better peak isolation. It is also known that \( P_a(k) \equiv 0 \mod 4 \) for all \( k \), for any binary sequence of period \( L = 2^n \), with \( n \geq 2 \).

Provided that in a binary De Bruijn sequence the number of 1’s equals the number of 0’s, it turns out that any binary De Bruijn sequence, when converted into a bipolar form, has a zero average autocorrelation. Fig. 6 shows the average autocorrelation profile computed over all the 2048 binary De Bruijn sequences of length 32. With the exception of the peak value at \( k = 0 \), the autocorrelation profile is symmetric with respect to the central value of the shift, \( k = 16 \). As a matter of fact, it is possible to prove that \( P_a(k) = P_a(L - k) \), for \( 0 \leq k \leq L - 1 \). So, when \( n \) increases, the autocorrelation profiles of the De Bruijn sequences will show many samples equal to 0, a symmetric distribution of the samples, and a reduced number of different positive and negative samples, as to give an average autocorrelation equal to 0.

Fig. 6. Average autocorrelation profile of binary de Bruijn sequences of length 32

![Average autocorrelation profile](image)

Fig. 7 shows the set of autocorrelation profiles of all the 2048 binary De Bruijn sequences of length 32, with the exception of the peak value (that is obviously 32 in all the cases), to improve figure readability. The symmetry of each graph and the presence of null values are easily recognized. The first \((n - 1)\) samples adjacent to the peak, that is located at \( k = 0 \), are always equal to 0, as discussed above. This is an important feature to improve the
detection capability of the autocorrelation peak, i.e. the matched filtering of the received sequence.

Fig. 7. Autocorrelation profiles of all the 2048 binary de Bruijn sequences of length 32 (peaks removed)

For the cross-correlation function of a pair of De Bruijn sequences \( a \) and \( b \) (\( a \neq b \)) of the same span \( n \) and length \( L \), the following bound holds:

\[-2^n \leq P_c(k) \leq 2^n - 4, \text{ for } 0 \leq k \leq L - 1,\]

where \( P_c(k) \) denotes the cross-correlation between the two sequences. Any couple of sequences including a De Bruijn sequence \( c \) and its complementary one \( c^* \), that is also a De Bruijn sequence and belongs to the same family, may provide a negative peak of cross-correlation equal to \(-2^n\) for a specific value of the shift \( k \), that motivates the lower bound on the cross-correlation profile. If we need to avoid this negative peak value of the cross-correlation, only half of the sequences in the set should be considered, by taking just one sequence in each couple \((c, c^*)\). The cross-correlation function of binary De Bruijn sequences shows symmetry properties similar to those exhibited by the autocorrelation function. All the possible cross-correlation values are integer multiple of 4.

**DS SS Chaotic Radar: Analysis and Simulation Results**

This section presents the peculiarities of the DS-SS radar and an algorithm developed to improve the related detection capability. Performance in terms of correct detection probability and false detection probability are discussed, by means of numerical simulations.

**DS/SS Radar Basics**

A DS/SS radar is basically an electromagnetic system for the detection and location of obstacles or targets, that operates by radiating energy into space and detecting the echo.
signal reflected from an obstacle, or target, in the detection area. In a DS/SS configuration, each radar uses a different spreading sequence, composed by \( N \) chips, that modulates the transmitted signal.

**Range to a Target**

The range to a target is determined by the time \( T_R \) it takes the radar signal to travel to the target and back. Electromagnetic energy in free space travels with the speed of light, which is \( c = 3 \times 10^8 \) m/s. Thus, the time for the signal to travel to a target located at a range \( R \) and get back to the radar is \( 2R/c \). The range to a target is then:

\[
R = \frac{cT_R}{2} \quad (9)
\]

With the range expressed in Kilometers, and \( T_R \) in microseconds, the range can be written as \( R(\text{km}) = 0.15 \cdot T_R(\text{μs}) \); in other words, each microsecond of round-trip travel corresponds to a distance of 150 meters. Therefore, in order to calculate the target distance \( R \), it is necessary to measure \( T_R \), which is evaluated in a DS radar through the correlation between the received spreading code and the reference spreading code.

**Maximum Unambiguous Range**

Once a spreading sequence is radiated into the space by a radar, a sufficient time must elapse to allow all echo signals get back to the radar receiver, before the next spreading sequence is transmitted. The rate at which sequences may be transmitted, therefore, is determined by the longest range at which targets are expected. In a LRR the maximum detectable range \( R_{\text{max}} \) is 150 meters, which corresponds to a maximum round-trip travel of one microsecond, as illustrated above. Thus, the transmitted signal will be modulated by a spreading sequence, periodically repeated, lasting one microsecond, in order to detect a vehicle without ambiguity.

By assuming that each sequence has a duration \( T = 1 \) μs and length \( N \), the chip duration defined as:

\[
T_c = \frac{T}{N} \quad (10)
\]

will determine the range resolution \( \Delta R \) as:

\[
\Delta R = \frac{c \cdot T_c}{2} \quad (11)
\]

**The Radar Equation**

The radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, target, and the environment. If the transmitter power \( P_t \) is radiated by an isotropic antenna, the power density at a distance \( R \) from the radar is equal to the radiated power divided by the surface area \( 4\pi R^2 \) of an imaginary sphere of radius \( R \):

\[ P_R = \frac{P_t}{4\pi R^2} \]
Power density at range $R$ from an isotropic antenna \( \frac{P}{4\pi R^2} \) (12)

Radar antennas, however, employ directive antennas, with gain $G$, to concentrate the radiated power in a particular direction. Therefore the power density can be written as:

Power density at range $R$ from a directive antenna \( \frac{PG}{4\pi R^2} \) (13)

The target intercepts a portion of the incident energy and radiates it in various directions. Only the power density re-radiated in the direction of the radar (the echo signal) is of interest. The radar cross section of the target determines the power density returned to the radar for a particular power density incident on the target; it is denoted by $\sigma$ and is often called RCS. The radar antenna captures a portion of the echo energy incident on it.

The received power is given as the product of the incident power density times the effective area $A_e$ of the receiving antenna. The received signal power $P_r$ is then:

\[
P_r = \frac{PGA_e\sigma}{(4\pi)^2 R^4}
\] (14)

The maximum range of a radar is the distance beyond which the target cannot be detected. It occurs when the received signal power $P_r$ just equals the minimum detectable signal.

By assuming that the RCS equals 1 for all the detected targets, and leaving $P_r$, $G$ and $A_e$ out, since they are the same for all the echoes, the received signal power of the i-th echo can be written as:

\[
P_{r,i} \propto \frac{1}{(4\pi)^2 R_i^4}
\] (15)

where $R_i$ is distance of the i-th target, whereas the amplitude of the i-th signal $A_{r,i}$ is proportional to:

\[
\frac{1}{4\pi^2 R_i^2}
\] (16)

Proposal of a Detection Algorithm for a Multi-user Radar Environment

A major difficulty, in automotive radar operation, is caused by the presence of several targets in the scanned region, which is responsible for the appearance of multiple echoes as well. Let us consider Fig.8: the radar on board of the leftmost car intercepts three vehicles, A, B and C, at distance $L_A = 40$ m, $L_B = 90$ m and $L_C = 140$ m, respectively. By assuming, for simplicity, a unitary cross section for each vehicle, the amplitude of each signal detected by the radar sensor after the round trip path is proportional to $1/L_i^2$, with $i = A, B, C$. Then the received signal:

\[
y(t) \propto \frac{1}{L_A^2} \delta(t - \tau_A) + \frac{1}{L_B^2} \delta(t - \tau_B) + \frac{1}{L_C^2} \delta(t - \tau_C)
\] (17)
consists of the superposition of three replicas (echoes) of the transmitted signal \( x(t) \) but with different delays \( \tau_i \), that the sensor system aims to estimate. Fig.9 shows the AC functions, for the example discussed, obtained by using Gold sequences of length \( N_G = 63 \) and \( N_G = 127 \). For better evidence, the results are plotted directly as a function of the distance \( L \). A peak is evident at \( L_A = 40 \) m, which is due to (and permits to detect the presence at this distance of) vehicle A. Correlation peaks for the other vehicles, instead, are rather small (because of their larger distance), and apparently masked in the figure.

Fig.8. Example of multiple targets in the radar operating range.

Fig.9. Auto-correlation for the case in Fig.8.

On the other hand, the radar system should determine also the distance from the other vehicles. Once detected the greatest peak, typically through the comparison with a threshold, the processor on board of the vehicle can subtract the detected sequence, with its own delay and amplitude, from the received signal, to eliminate the dominant contribution and discover the others. A new correlation is then computed on the cleaned signal, searching for a new AC peak. As an example, Fig.10 shows the second correlation computed, which evidences, for both the considered sequence lengths, a peak at \( L_B = 90 \) m, which is due to (and permits to detect the presence at this distance of) vehicle B. The procedure herein described is iterated, until no peak above the threshold is found. Another obvious criterion to stop the procedure consists in not accepting, from the algorithm, peak positions that are at a shorter distance than those already found. It is important to stress that signal processing required for these operations is very fast, so that a large number of peaks can be detected in a very short time. The position of the threshold should be optimized, as a function of the number and distance of the vehicles that the system should detect at each scan; too much small
thresholds should be avoided, as they can cause a large number of false detections. On the other hand, also the risk of false detection is strictly related to the correlation properties of the sequences used, thus implying an in depth study.

Fig. 10. Auto-correlation for the case in Fig.8, after elimination of the effect of the dominant term.

Continuing the example in Fig.8, the algorithm has been tested over 20 Gold sequences (for each length) and assuming a threshold equal to 2.5 times the correlation standard deviation; in all cases, vehicles A, B and C have been detected, and their distance correctly estimated. Then, Gold sequences seem adequate to solve the problem of intercepting vehicles that are moving along the same direction of the tagged car.

Actually, in this preliminary example, we have neglected the effect of the interfering signal produced by other radar equipped vehicles, whose echoes may disturb reception. For example, vehicle A in Fig.8 may be also equipped with a radar: its signal is then reflected from vehicle B and adds to the total signal received by the tagged sensor. Through simulation, we verified that 6 of the selected Gold sequences with length \( N_G = 127 \) become unable to detect the presence of vehicle B because of the interfering signal.

The impact of an interfering signal is, obviously, much stronger when originating from a vehicle that moves in the opposite direction (from right to left, in Fig.8). An example is shown in Fig.11. The interfering radar, at distance \( L_I \), obviously uses a spreading sequence different from that of the radar under test. However, the amplitude of the interfering signal is proportional to \( 1/L_I \), and a serious risk exists that it masks all the other signals. In practice, although the interfering signal weakly cross-correlates with the useful emitted signal, the more favorable attenuation law can “blind” the radar. This occurs when the ratio between the useful signal and the interfering signal becomes smaller than a minimum value (for example, Signal-to-Interference Ratio, SIR < 0.03).
It should be noticed that such very unfavorable situations take place only when the interfering signal is received in the direction of maximum antenna gain. Because of the relative movement of the antennas, whose beam is electronically controlled, this happens only for a fraction of time in a vehicular scenario. Simple systems can be designed, for example based on iterated measurements, which are able to overcome, in real time, the criticalities. On the other hand, for larger SIR values, the ability to detect a vehicle by a single measurement depends on the correlation properties of the spreading signal adopted. Numerical examples will be provided, after having introduced possible usage of chaotic signals, thus realizing a comparison between the two solutions.

Besides the probability of correct detection, we have to consider the probability of false detection, i.e. the radar detects an obstacle that actually is not present. A false detection occurs when a correlation value greater than the fixed threshold does not correspond to a vehicle echo but, instead, to an interfering signal generated by another radar. So, too much small thresholds should be avoided, as they can cause a large number of false detections. The threshold required to minimize false detections depends on the SIR and the sequence length adopted. For example, simulations show that for any SIR $\geq 0.03$, the probability of false detection is almost zero by assuming a threshold 3 times larger than the AC standard deviation, when using Gold sequences of length $N_G = 63$. A threshold 2.5 times larger than the AC standard deviation, when using Gold sequences of length $N_G = 127$, is needed. It is clear that the selection of the most suitable threshold value is a trade-off between the need to have a high probability of correct detection (that increases with threshold reduction), and a low probability of false detection (that shows an opposite behavior).

**Performance Improvement by Using Chaotic Spreading Signals**

The maximization of correct detection probability, and minimization of false detection probability, are conditioned on the adoption of spreading sequences with good correlation properties. Gold codes can ensure acceptable performance, but the correlation properties, and other features of the system, can be improved by replacing Gold signals with chaotic signals.

In this work, focus is on the use of the chaotic evolution resulting from the solutions to the Lorenz system (see Eq.2). To compare the performance of different spreading sequences, an important requirement concerns the need to assume almost the same
bandwidth expansion factor, otherwise the comparison would be unfair. Bandwidth is
determined on a power content basis, i.e. as the frequency range including a given
percentage (95% for example) of the total power. In the case of Lorenz chaotic signals,
the bandwidth factor does not result “naturally” from the solution of the system. The
system is approximated by an equivalent discrete form:
\[
\begin{align*}
  u_{t+1} &= u_t + b \cdot \sigma \cdot (v_t - u_t) \\
  v_{t+1} &= v_t + b \cdot (r \cdot u_t - v_t - 20 \cdot u_t \cdot w_t) \\
  w_{t+1} &= w_t + b \cdot (5 \cdot u_t \cdot v_t - b \cdot w_t)
\end{align*}
\]

for a given set of parameters and initial conditions, signal evolution depends on the
discretization step, and on the sampling period. A possible procedure consists in fixing
the sequence duration \(T\), and then assuming a number of chaotic samples that ensure
(approximately) the same bandwidth expansion factor of the competing Gold sequence.
Therefore, the number of samples in a chaotic sequence derived from the Lorenz system,
\(N_L\), has the same meaning of the length \(N_G\) of a Gold sequence. Maintaining the duration
\(T\) and extracting, at a given rate, \(N_L = 124, 252\) and \(508\) chaotic samples, we have almost
the same 95% power bandwidth occupancy of Gold sequences with \(N_G = 31, 63\) and \(127\),
respectively. About the dispersion of the CC and AC values around the mean (i.e., the
standard deviation), we have calculated the AC and CC standard deviation for 8 Gold
sequences and 8 chaotic segments, at a parity of bandwidth occupancy. The various cases
have been denoted by “Gold \(N\)” and “Lorenz \(N\)” respectively. The results in Table 3 show
that the values for the chaotic segments are always smaller.

<table>
<thead>
<tr>
<th></th>
<th>CC Standard Deviation</th>
<th>AC Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold 31</td>
<td>0.178</td>
<td>0.177</td>
</tr>
<tr>
<td>Lorenz 124</td>
<td>0.090</td>
<td>0.083</td>
</tr>
<tr>
<td>Gold 63</td>
<td>0.123</td>
<td>0.125</td>
</tr>
<tr>
<td>Lorenz 252</td>
<td>0.063</td>
<td>0.059</td>
</tr>
<tr>
<td>Gold 127</td>
<td>0.087</td>
<td>0.088</td>
</tr>
<tr>
<td>Lorenz 508</td>
<td>0.044</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Generally speaking, it is possible to verify that the distribution of the AC and CC values
is more similar to a Gaussian distribution with zero mean. Since the interfering effect is a
decreasing function of the standard deviation, and is more favorable in the case of a
Gaussian-like distribution, it is easy to think that the adoption of the chaotic solution in
the DS-SS radar should enable more accurate and robust detections.

The potential advantages offered by chaotic sequences are manifold. First of all, chaotic
sequences exhibit \(N_L > N_G\); equivalently, the “chip duration” \(T_C\) of the spreading
sequence is smaller. As in a radar system the value of \(T_C\) determines the resolution \(\Delta R\),
according with Eq. 11, by using chaotic codes it is possible to have more precise distance
estimations. As shown in Table 4, by using the chaotic codes, a four times smaller range resolution is achieved.

Table 4. Range resolution.

<table>
<thead>
<tr>
<th></th>
<th>Gold 31</th>
<th>Gold 63</th>
<th>Gold 127</th>
<th>Lorenz 124</th>
<th>Lorenz 252</th>
<th>Lorenz 508</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔR</td>
<td>4.84 m</td>
<td>2.38 m</td>
<td>1.18 m</td>
<td>1.21 m</td>
<td>0.6 m</td>
<td>0.3 m</td>
</tr>
</tbody>
</table>

The most important benefit of using chaotic signals in place of Gold sequences is in the possibility to increase the probability of correct detection and to reduce the probability of false detection, at a parity of the other parameters. A first example is given in Table 5: for a scenario of the type reported in Fig.11, where an interfering radar is present, we computed the percentage of correct detections for the case of Gold sequences and chaotic (Lorenz) sequences, with different length, and by choosing different threshold values ($T_{th}$*standard deviation of the computed correlation), to detect the correlation peaks. Results refer to the most unfavorable condition, when the interfering radar is emitting its signal exactly in the direction of the receiving antenna maximum gain. Besides the codes mentioned above, a chaotic sequence with $N_{L} = 1020$ is also considered, that, as expected, exhibits the best performance. From Table 5 we see that, at a parity of the bandwidth expansion factor, the chaotic signals generally ensure higher percentages of correct detection. On the other hand, the detection ability improves for increasing bandwidth expansion; in particular, the chaotic signal with $N_{L} = 1020$ ensures 100% of successful detection, even for a SIR value as low as 0.23. Conversely, for SIR = 0.03, the radar is totally blind, independently of the spreading sequence adopted; as discussed previously, however, this “out-of-service” condition is limited to a very short time, because of the vehicle/antenna movements, and can be compensated through suitable signal processing procedures.

Besides the probability of correct detection, mentioned above, also the probability of false detection shall be taken into account, i.e. the probability that the radar detects an obstacle in the scanned area which is actually not present. In particular, a false detection is generated when there is a correlation peak above the threshold that does not correspond to a vehicle. Table 6 reports a number of examples in this sense. Results have been obtained by repeating simulation with (a large number of) different pairs of spreading sequences for the tagged and the interfering radars, and enumerating the number of cases when false detections were revealed, versus the total number of simulations. A too small threshold value should be avoided, since it can cause a large number of false detections. Moreover, there is not a threshold value suitable for all kind of sequences, but the choice is related to the type of sequences adopted, and it is a trade-off between the need to have a high probability of correct detection, and a reduced occurrence of false detections.
Table 5. Percentages of correct detections as a function of the SIR for Gold and Lorenz signals.

<table>
<thead>
<tr>
<th>Threshold (SIR)</th>
<th>Gold 63</th>
<th>Lorenz 252</th>
<th>Gold 127</th>
<th>Lorenz 508</th>
<th>Lorenz 1020</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>80%</td>
<td>80%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>10%</td>
<td>50%</td>
<td>70%</td>
<td>45%</td>
<td>15%</td>
</tr>
<tr>
<td>3.5</td>
<td>10%</td>
<td>5%</td>
<td>15%</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>4</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 6. Percentages of false detection as a function of the SIR for Lorenz signals.

<table>
<thead>
<tr>
<th>Threshold (SIR)</th>
<th>Lorenz 252</th>
<th>Lorenz 508</th>
<th>Lorenz 1020</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>3.5</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>3.5</td>
<td>10%</td>
<td>40%</td>
<td>45%</td>
</tr>
<tr>
<td>4</td>
<td>10%</td>
<td>45%</td>
<td>50%</td>
</tr>
</tbody>
</table>

When adopting chaotic sequences of length N=508 for small values of the SIR, the threshold that guarantees the higher correct detection percentages also determines the higher rate of false detections, as shown again in Fig.12.

Table 7 shows the threshold values chosen for the simulations presented in the following part of this chapter, as the best trade-off between the need to have a high probability of correct detection, and a low occurrence of false detections.

Table 7. Optimized values to set the threshold.

<table>
<thead>
<tr>
<th>Threshold (SIR)</th>
<th>Gold 31</th>
<th>Lorenz 124</th>
<th>Gold 63</th>
<th>Lorenz 252</th>
<th>Gold 127</th>
<th>Lorenz 508</th>
<th>Lorenz 1020</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
<td>3.5%</td>
<td>3.5%</td>
<td>3.5%</td>
<td>3.5%</td>
</tr>
</tbody>
</table>
Fig. 12. Percentages of sequences providing false alarm (a) and percentages of correct detection (b) as a function of the SIR for Lorenz 508 signals for different values of the threshold.

Simulation Results

In this Section, simulation results obtained in different scenarios are presented. The algorithm previously described has been tested in several realistic multi-target scenarios.
The proposed road environment is again of the type shown in Fig.11, i.e. a four-lane two ways road, each lane being 4 m large. The radar under test, and any other interfering radar in the explored region, scans an area 150 m long and 30° wide, by 15 steps of 2° each. The availability of radars with very narrow radiation diagrams reduces the risk of blinding; when the signal from the interfering radar is detected along a direction different from that of maximum gain, the interfering effect is reduced. The more the misalignment between the two radar beams, the less the impact on the correct detection capability. Considering also that the relative positions of the vehicles proceeding in opposite directions change continuously, and rapidly, the blinding effect, whose duration is limited in time, can be compensated by on board processing. However, investigation of the most critical operation conditions remains a valuable task: to verify that the radar has a good response even when the detected signal is strongly degraded, gives some guarantee that the system will be able to react in time, by correlating the information collected over multiple scans, mostly in terms of differences emerged.

A pictorial way to represent the result of a radar scan is to use 2D or 3D maps that, by plotting the correlation values as a function of the spatial coordinates, permit to localize directly the position of the detected vehicles. It is important to stress again that the correlation peaks are extracted one by one, according to the algorithm presented above. For this reason, as will be shown afterwards, they appear as normalized graphs, where all the peaks identifying vehicles have almost the same altitude. At the same time, it is also important to remind the fundamental role of the threshold optimization, as false peaks can artificially result from the algorithm if the threshold value is not correctly set.

**First set of Simulation Results: Ideal Antenna**

In the first implementation it is reasonable to think that only a small percentage of cars will be equipped with automotive radars; for this reason, in our first simulated scenario (see Fig.13), we have the presence of only a few systems of this type; Fig.13, for example, is quite similar to Fig.11, but there are two interfering radars.

Fig.13. Example with two interfering radars.

In the first simulation, Gold sequences with length $N_G = 31, 63$ and $127$ respectively, and Lorenz chaotic sequences with length $N_L = 124, 252, 508$ and $1020$, are tested.
First of all, coherently with the analysis above, it is important to remark the importance of threshold optimization.

Fig. reports a 2D map obtained by using chaotic sequences with $N_L = 124$, for the scenario of Fig.13, and assuming a threshold 2 times larger than the correlation standard deviation. In this 2D map (and more generally in all the maps shown below), the color intensity is related to the correlation peaks amplitude: the red spots correspond to higher peaks, whereas the blue color corresponds to almost null correlation values. Moreover, a mask, representing the vehicular scenario, is overlaid to the map, to immediately locate the vehicles detected and those remaining undetected. Because of the small threshold, an impressively high number of spurious spots (i.e. false detections) appear in Fig.14, that make the map quite useless.

Fig.14. 2D map for the scenario in Fig.13, using chaotic sequences with $N_L = 124$ and not optimized threshold.

Fig.14 shows the 3D map corresponding to the simulation in Fig.13. The correlation peaks should correspond to the vehicles that have been detected. But in this case the high number of visible peaks is due to the interfering radars, and the threshold that is too low; therefore, these peaks correspond to false detections, and this enforces the importance of selecting an optimized threshold value.

Fig.15. Error! Reference source not found. 3D map for the scenario in Fig.13, using chaotic sequences with $N_L = 124$ and not optimized threshold.

By choosing the values reported in Table 7, we can minimize the number of false detections, as shown in Figs. 16-21.
Fig.16. 3D map for the scenario in Fig.13, using chaotic sequences with $N_L = 124$ and the optimized threshold.

Fig.17. 3D map for the scenario in Fig.13, using Gold sequences with $N_G = 31$.

Fig.18. 3D map for the scenario in Fig.13, using chaotic sequences with $N_L = 252$.

Fig.19. 3D map for the scenario in Fig.13 using Gold sequences with $N_G = 63$. 
The set of simulation results discussed above refers to the almost ideal case of radar antennas with negligible side lobes. This implies that, for each interfering radar, the interference is maximum at the scan position (1 out of 15) for which the radar under test is directed towards the interfering one, and null outside. Reciprocally, the worst condition happens when the interfering signal emits along the direction of maximum gain for the radar under test. Actually, an upper bound for the probability this event occurs can be roughly estimated as $(1/15)^2$. Figures 16-21 evidence the existence of a blinding zone (clearly indicated in Fig. 22), caused by the interfering radar; in practice the system recognizes, at the right distance, 7 out of 10 vehicles, while the cars equipped with the interfering radars, and the one behind the second of them, are not detected. In this scenario, all the tested sequences actually detect the same seven vehicles, they cannot detect the vehicles equipped with the interfering radar and the vehicle behind them. Therefore, the performance of the two kinds of sequences is almost equivalent in this idealized scenario.
Second Simulation Set: Antenna with Side Lobes

Let’s now remove the hypothesis on the antenna, and assume that the interference signal is also captured, though attenuated, along directions different from the one corresponding to maximum gain (because of the antenna side lobes). In this case, it is possible to verify that the chaotic sequences outperform the Gold sequences, at a parity of the bandwidth expansion factor, which is coherent with the previous analysis.

Let us consider again the road environment in Fig.11, that includes only one interfering radar; let us suppose that the radars are aligned, but the interfering signal is not zero also in directions different from that of maximum gain. In details, the interfering signal is:
- attenuated by 10 dB in the direction ± 2° away from the maximum,
- attenuated by 20 dB in the direction ± 4° away from the maximum,
- attenuated by 30 dB in the directions that differ by more than 4° (in modulus) from the maximum.

This situation, that however still represents a case of blinding, is less favorable than the previous one (even though the number of interfering radars has been reduced), because of the increased interfering signal. The 3D map of the correlation function is shown in Fig.23: in this simulation, Lorenz chaotic sequences with (almost) the same 90% power bandwidth occupancy of Gold sequences with $N_G = 31$ are considered, i.e. a less conservative (than before) bandwidth definition; under this assumption, $N_L = 200$ must be set for a fair comparison.

Fig.23. 3D map for the scenario in Fig.11, using chaotic sequences.

There are 5 vehicles clearly detected through high correlation peaks, while the lower peak in the centre of the map corresponds to the vehicle at distance 90 meters along lane 2; because of the value set for the threshold, that is necessary to avoid false detections, this peak is difficult to identify. The 3D map obtained when Gold sequences are applied is significantly worse; this is shown in Fig.24. Only 4 vehicles are detected, the blind zone extension is much wider and, even more evident, the map is more contaminated than before, so that distinguishing the peaks is more difficult.
Third Simulation Set

Let us consider the road environment depicted in Fig. 25, where a single interfering radar is present, which is very close to the useful radar; let us suppose that the radars are aligned, but the interfering signal is not zero also in directions different from that of maximum gain, in the same conditions assumed before.

This situation, again dealing with a blinding event, is less favorable than the previous one, because of the reduced distance of the interfering radar. In this simulation, only chaotic codes are evaluated, in particular their ability to fight the interfering noise by increasing the sequence length, and therefore the bandwidth. Figures 26-28 represent the 3D maps obtained by using Lorenz sequences of length $N_L = 252$, 508 and 1020, respectively.
The maps show that, by increasing the bandwidth, it is possible to remarkably increase the detection ability of the radar; in fact the number of vehicles detected passes from 3 to 6, by increasing the sequence length from $N_L = 252$ to 1020.

Finally, in Fig. 29 the same scenario, but without interfering radars, is considered: in this favorable situation all peaks are correctly detected (the one referred to the farthest vehicle, which is the last one to be detected through the identification iterative algorithm, is smaller). This is the map we can obtain, even in the presence of interfering radars, during the time intervals when the interfering signal is weak, because of the misalignment of the antenna beams.
It is expected this condition occurs for most of the time, thus preserving, on average, the correct behavior. The worst conditions discussed before result critical to be overcome through the combination algorithms applied by the on board processor. Reducing the impact of the criticalities, as permitted by the proposed set of chaotic spreading sequences, simplifies the computational complexity, and eventually makes processing more feasible and reliable.

PRELIMINARY RESULTS ABOUT DE BRUIJN SEQUENCES FOR DS-SS RADARS

To estimate the applicability of De Bruijn sequences in the DS-SS radar context, we start from the evaluation of their auto-correlation properties, and the comparison with M- and Gold sequences of similar length. Given the complexity of the generation process for the De Bruijn sequences, we limit the discussion to a subset of De Bruijn sequences of period \( N = 32 \) (\( n = 5 \)). This choice allows a fair comparison with M-sequences and Gold sequences of length 31.

The auto-correlation profiles of the M-sequences and Gold sequences are well known. A De Bruijn sequence exhibits a satisfactory behavior, especially if compared to a Gold (\( N_G = 31 \)) one: for \( k \neq 0 \), the normalized autocorrelation values are, on average, smaller than those assumed by the Gold sequence, as shown in Fig.30.

Sequences with good auto-correlation properties are useful in several contexts; the goodness of a sequence may be expressed through properly defined metrics such as the Root-Mean-Square (RMS) sidelobe level of the auto-correlation function of a sequence \( a \), defined as:

\[
P_{A,RMS}^{(a)} = \frac{1}{N-1} \sum_{i=1}^{N-1} \left( P_{A}^{(i)}(a) \right)^2 
\]

Fig.30. Normalized auto-correlation curves for three sample sequences: Gold and M-sequence of length 31, and a De Bruijn sequence of length 32.

The results obtained for the sequences of interest are summarized in Table 8. The M-sequences provide the best behavior in terms of metric minimization, however, they also
provide the smallest set of available sequences. It is worth noting that the whole family of 2048 De Bruijn sequences of span \( n = 5 \) provides a lower expected value of \( P_{a,RMS}^{(a)} \) than the family of 33 different Gold \( N_G=31 \) sequences. It is possible to select, by numerical verification, a subset of the De Bruijn sequences of span 5 which provide closer agreement to random sequences, with respect to M- and Gold sequences, in terms of \( P_{a,RMS}^{(a)} \). Cross-correlation properties of the sequences were also studied. The De Bruijn sequences show the highest maximum absolute value of the cross-correlation, a zero average value, and a standard deviation that is very close to the one provided by the other sets of sequences. The maximum absolute value of the cross-correlation for the De Bruijn sequences, which equals the peak auto-correlation value, is due to the coexistence, in a complete family of span \( n \) sequences, of different sequences, and their corresponding complementary ones. If the complementary sequences are excluded from the set, the maximum absolute cross-correlation value is reduced, without affecting the cardinality of the set, which remains huge. In the vehicular radar application context, this consideration would motivate the selection of some De Bruijn sequences, within a whole set, to use as radar signatures, in order to lower the maximum absolute cross-correlation as much as possible, and reduce the probability of false detection.

Table 8. Expected value of the RMS sidelobe level of the auto-correlation function, for the test sequences.

<table>
<thead>
<tr>
<th></th>
<th>Avg. ( P_{a,RMS}^{(a)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-sequences</td>
<td>1</td>
</tr>
<tr>
<td>Gold ( N_G=31 )</td>
<td>5.34</td>
</tr>
<tr>
<td>De Bruijn</td>
<td>4.67</td>
</tr>
</tbody>
</table>

**CONCLUSION**

Chaotic signals are of potential interest for application in long-range automotive radars based on DS-SS. Their performance is often better than that of Gold codes with an (almost) identical bandwidth expansion factor. On the other hand, they exhibit the important feature, for these kinds of applications, of maintaining favorable correlation properties over a large set of spreading sequences. These conclusions have been drawn by considering typical, although necessarily simplified, road scenarios. The major problem in these kinds of systems concerns the blinding effect due to an interfering radar proceeding in the opposite direction; SS systems offer better performance from this point of view than that ensured by more classical schemes. The chaotic solution does not seem much more complicated to implement, and it is even more robust against interference.

In fixing the performance, an important role is played by the correct positioning of the threshold against which the correlation function is compared. We have suggested a “rule of thumb,” which is easy to implement and yielded very good performance in the considered situations. However, the design of effective algorithms that are able to optimize the threshold value in any operation condition remains a valuable open issue.
The behavior exhibited by the set of De Bruijn sequences tested, in terms of auto- and cross-correlation properties, is encouraging, and motivates the efforts aimed at generating the complete set of De Bruijn sequences of span n, for even greater values of n, and performing an exhaustive evaluation of their properties.

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